# Estimation Tasks – Questions and Answers

## Problem 1: MLE for Bernoulli Distribution

A coin is flipped n = 10 times, and we observe x = 7 heads. Assume the coin follows a Bernoulli distribution with parameter p.

1. Likelihood Function: L(p) = p^x (1 - p)^(n - x) = p^7 (1 - p)^3

2. Log-Likelihood Function: ln L(p) = 7 ln(p) + 3 ln(1 - p)

3. MLE: Differentiate and set derivative = 0 → p̂ = x / n = 7 / 10 = 0.7

## Problem 2: MLE for Binomial Distribution

A factory produces light bulbs, and in a sample of n = 20 bulbs, k = 15 are defective.

1. Likelihood Function: L(p) = C(n, k) p^k (1 - p)^(n - k) = C(20,15)p^15(1 - p)^5

2. Log-Likelihood Function: ln L(p) = 15 ln(p) + 5 ln(1 - p) + constant

3. MLE: p̂ = k / n = 15 / 20 = 0.75

## Problem 3: MLE for Poisson Distribution

The number of customers arriving per hour follows Poisson(λ). Suppose 5 customers arrive in an hour.

1. Likelihood Function: L(λ) = (e^(-λ) λ^x) / x! = e^(-λ) λ^5 / 5!

2. Log-Likelihood Function: ln L(λ) = -λ + 5 ln(λ) + constant

3. MLE: d/dλ ln L = -1 + 5/λ = 0 ⇒ λ̂ = 5

## Problem 4: MLE for Normal Distribution

Given Xi ∼ N(µ, σ²).

1. Likelihood Function: L(µ, σ²) = (1 / (2πσ²)^(n/2)) exp[-Σ(xi - µ)² / (2σ²)]

2. MLE for µ (σ² known): µ̂ = X̄

3. MLE for σ²: σ̂² = (1/n) Σ(xi - X̄)²

## Problem 1: MOM for Bernoulli Distribution

Observed mean X̄ = 0.6.

E[X] = p → p̂ = X̄ = 0.6

## Problem 2: MOM for Binomial Distribution

For X ∼ Bin(n, p), E[X] = np → p̂ = X̄ / n = 5 / 30 = 1/6 ≈ 0.167

## Problem 3: MOM for Poisson Distribution

E[X] = λ → λ̂ = X̄ = 4.5

## Problem 4: MOM for Normal Distribution

E[X] = µ, Var(X) = σ² → µ̂ = X̄ = 10, σ̂² = S² = 4